

Math (Science)	Group-II	Paper-I
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

SECTION-I

2. Write short answers to any SIX (6) questions: 12

(i) Define square matrix, give example.

Ans A matrix is called a square matrix if its number of rows is equal to its number of columns.

Example:

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix} \quad \text{and } C = [3]$$

(ii) If $C = [1 \quad -1 \quad 2]$, then find: $3C = ?$

Ans

$$C = [1 \quad -1 \quad 2]$$

$$3C = 3 \cdot [1 \quad -1 \quad 2]$$

$$= [3 \quad -3 \quad 6]$$

(iii) Write the real and imaginary part of number: $(-3i + 2)$.

Ans

Let $z = -3i + 2$

Real part = 2

Imaginary part = -3

(iv) Simplify: $\frac{4(3)^n}{3^{n+1} - 3^n}$

Ans

$$= \frac{4 \cdot 3^n}{3^{n+1} - 3^n}$$

$$= \frac{4 \cdot 3^n}{3^n \cdot 3^1 - 3^n}$$

$$= \frac{4 \cdot 3^n}{3^n (3 - 1)} = \frac{4 \cdot 3^n}{3^n \cdot 2}$$

$$= 2.$$

(v) Find the value of x if: $\log_x 64 = 2$.

Ans $\log_x 64 = 2$

Write in exponent form.

$$\begin{aligned}(x)^2 &= 64 \\ (x)^2 &= (8)^2 \\ (x^2)^{1/2} &= (8^2)^{1/2} \\ x &= 8\end{aligned}$$

(vi) Write in the form of single logarithm:

$$2 \log x - 3 \log y$$

Ans \rightarrow
$$\begin{aligned}2 \log x - 3 \log y \\ = \log x^2 - \log y^3 \\ = \log \frac{x^2}{y^3}\end{aligned}$$

(vii) Rationalize the denominator:

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Ans \rightarrow
$$\begin{aligned}\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \\ = \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-1} \\ = \frac{(\sqrt{3}-1)^2}{3-1} \\ = \frac{(\sqrt{3}-1)^2}{2}\end{aligned}$$

(viii) Simplify: $\left(\sqrt{2} + \frac{1}{\sqrt{3}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{3}}\right)$

Ans \rightarrow
$$\begin{aligned}\left(\sqrt{2} + \frac{1}{\sqrt{3}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{3}}\right) \\ = (\sqrt{2})^2 - \left(\frac{1}{\sqrt{3}}\right)^2 \\ = 2 - \frac{1}{3} = \frac{6-1}{3} \\ = \frac{5}{3}\end{aligned}$$

(ix) Factorize: $30x^2 + 7x - 15$.

Ans \rightarrow
$$\begin{aligned}30x^2 + 7x - 15 \\ = 30x^2 + 25x - 18x - 15 \\ = 5x(6x + 5) - 3(6x + 5) \\ = (6x + 5)(5x - 3)\end{aligned}$$

3. Write short answers to any SIX (6) questions: 12

i) Find square root by factorization:

$$4x^2 - 12x + 9$$

Ans $4x^2 - 12x + 9$
 $= (2x)^2 - 2(2x)(3) + (3)^2$
 $= (2x - 3)^2$
Square root $= \sqrt{(2x - 3)^2}$
 $= \pm (2x - 3)$

(ii) Define linear equation and write down its standard form.

Ans A linear equation in one variable x (occurring to the first degree) is an equation of the form.

Example:

$$ax + b = 0 \text{ where } a, b \in \mathbb{R} \text{ and } a \neq 0.$$

(iii) Solve the equation: $|3x - 5| = 4$

Ans $|3x - 5| = 4$
 $3x - 5 = \pm 4$

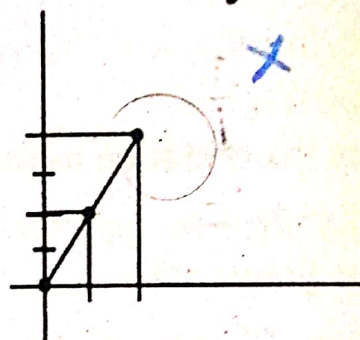
$3x - 5 = 4$	$3x - 5 = -4$
$3x = 4 + 5$	$3x = -4 + 5$
$3x = 9$	$3x = 1$
$x = \frac{9}{3} = 3$	$x = \frac{1}{3}$

S.S = $\{3, \frac{1}{3}\}$

(iv) Draw the graph of the equation $2x - y = 0$.

Ans $-y = -2x$
 $y = 2x$

Put $x = 0, 1, 2$
 $y = 0, 2, 4$
 $(0, 0), (1, 2), (2, 4)$



(v) Does the point $P(-1, 1)$ lie on the line

$2x - y + 1 = 0$

Ans $2x - y + 1 = 0$
 $P(-1, 1)$
 $2(-1) - 1 + 1 = 0$
 $-2 = 0$

Hence point does not lie on the line.

- (vi) Find the distance between the points $A(3, -11)$, $B(3, -4)$

Ans $A(3, -11)$, $B(3, -4)$

$$d = |AB| = \sqrt{(3 - 3)^2 + (-4 + 11)^2} = \sqrt{(0)^2 + (7)^2}$$

$$d = \sqrt{0 + 49}$$

$$d = 7$$

- (vii) Find the mid-point of the line segment joining the points $A(-8, 1)$ and $B(6, 1)$.

Ans $A(-8, 1)$, $B(6, 1)$

$$\text{Mid-point} = \left(\frac{-8 + 6}{2}, \frac{1 + 1}{2} \right)$$

$$= (-1, 1)$$

- (viii) What are you meant by $S.A.A. \cong S.A.A.$?

Ans In any correspondence of two triangles, if one side and any two angles of one triangle are the corresponding side and angles of the other, the two triangles are congruent ($A.S.A. \cong A.S.A.$)

- (ix) What are you meant by the point of trisection of a median?

Ans The point which divides medians in $1 : 2$ or $2 : 1$ is known as point of trisection of median.

4. Write short answers to any SIX (6) questions:

- (i) Define bisector of an angle.

Ans Angle bisector is the way which divides an angle into two equal parts.

- (ii) What is the distance between a line and a point lying on it?

Ans The distance between a line and a point lying on it is zero.

- (iii) Define proportion.

Ans Equality of two ratios is defined as the proportion. If $a : b = c : d$, then a, b, c and d are said to be a proportion.

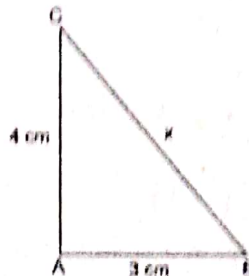
- (iv) Define similar triangles.

Ans Two (or more) triangles are called similar (symbol \sim) if they are equiangular and measures of their corresponding sides are proportional.

(v) Verify that the triangle having the following measures is a right angled when $a = 9$ cm, $b = 12$ cm, $c = 15$ cm.

Ans $c = 15$ cm, $b = 12$ cm, $a = 9$ cm
 $c^2 = (15)^2 = 225$
 $b^2 + a^2 = (12)^2 + (9)^2 = 144 + 81$
 $= 225$
 $c^2 = b^2 + a^2$ Proved.

(vi) Find the value of x from the following $\triangle ABC$:



Ans From fig.

$$|BC|^2 = |AB|^2 + |CA|^2$$
$$x^2 = (3)^2 + (4)^2$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = 5 \text{ cm}$$

(vii) Define area of a figure.

Ans The area enclosed by the boundary of the figure is called area of a figure.

(viii) Define orthocenter of a triangle.

Ans The point of concurrency of the three altitudes of a triangle is called its orthocenter.

(ix) The area of a parallelogram is equal to that of rectangle on the same base and having same altitude.

Ans Parallelograms on equal bases and having the same (or equal) altitude are equal in area.

SECTION-II

NOTE: Attempt any Three (3) questions. But question No. 9 is compulsory.

Q.5.(a) Solve the linear equations by Cramer's rule. (4)

$$4x + y = 9 ; \quad -3x - y = -5$$

Ans $4x + y = 9$
 $-3x - y = -5$

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$A X = B$$

$$|A| = \begin{vmatrix} 4 & 1 \\ -3 & -1 \end{vmatrix} = (4)(-1) - (-3)(1) \\ = -4 + 3 = -1$$

$$A_x = \begin{vmatrix} 9 & 1 \\ -5 & -1 \end{vmatrix} = -9 - (-5)$$

$$A_x = -9 + 5 = -4$$

$$A_y = \begin{vmatrix} 4 & 9 \\ -3 & -5 \end{vmatrix} \\ = (4)(-5) - (9)(-3) \\ = -20 + 27 = 7$$

$$x = \frac{A_x}{|A|}$$

$$x = \frac{-4}{-1} = 4 = \frac{A_y}{|A|} = \frac{7}{-1}$$

$$x = 4$$

$$y = -7$$

(b) Simplify: $\frac{2^{1/3} \times (27)^{1/3} \times (60)^{1/2}}{(180)^{1/2} \times (4)^{-1/3} \times (9)^{1/4}}$

Ans $\frac{2^{1/3} \times (27)^{1/3} \times (60)^{1/2}}{(180)^{1/2} \times (4)^{-1/3} \times (9)^{1/4}}$

$$= \frac{2^{1/3} \times (3^3)^{1/3} \times (2^2 \times 3 \times 5)^{1/2}}{(2^2 \times 3^2 \times 5)^{1/2} \times (2^2)^{-1/3} \times (3^2)^{1/4}}$$

$$= \frac{2^{1/3} \times 3^1 \times (2^2)^{1/2} \times 3^{1/2} \times 5^{1/2}}{(2^2)^{1/2} \times (3^2)^{1/2} \times 5^{1/2} \times 2^{-2/3} \times 3^{1/2}}$$

$$= \frac{2^{1/3} \times 3^1 \times 2^1}{2^1 \times 3^1 \times 2^{-2/3}}$$

$$= 2^{1/3} \times 2^{2/3}$$

$$= 2^{1/3 + 2/3} = 2^{1/3 + 2/3}$$

$$= 2^{3/3} = 2$$

Q.6.(a) Find the value by use of logarithm:

(4)

$${}^5\sqrt{2.709} \times {}^7\sqrt{1.239}$$

Ans Let $x = {}^5\sqrt{2.709} \times {}^7\sqrt{1.239}$

$$\log x = \log (2.709)^{1/5} \times (1.239)^{1/7}$$

$$\log x = \log (2.709)^{1/5} + \log (1.239)^{1/7}$$

$$= \frac{1}{5} \log (2.709) + \frac{1}{7} \log 1.239 = 0.0866 + 0.0133$$

$$\log x = 0.0999$$

$$x = \text{Antilog } (0.0999)$$

$$x = 1.2586$$

(b) If $5x - 6y = 13$ and $xy = 6$, then find the value of $125x^3 - 216y^3$. (4)

Ans $5x - 6y = 13$

$$xy = 6$$

$$125x^3 - 216y^3 = ?$$

$$(5x - 6y)^3 = (13)^3$$

$$(5x)^3 - (6y)^3 - 3(5x)(6y)(5x - 6y) = 2197$$

$$125x^3 - 216y^3 - 90xy(13) = 2197$$

$$125x^3 - 216y^3 - 90(6)(13) = 2197$$

$$125x^3 - 216y^3 - 7020 = 2197$$

$$125x^3 - 216y^3 = 2197 + 7020$$

$$125x^3 - 216y^3 = 9217$$

Q.7.(a) Factorize:

(3)

$$(x + 2)(x + 3)(x + 4)(x + 5) - 15$$

Ans $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

$$= \{(x + 2)(x + 5)\} \{(x + 3)(x + 4)\} - 15$$

$$= \{x^2 + 5x + 2x + 10\} \{x^2 + 4x + 3x + 12\} - 15$$

$$= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15$$

$$\text{Let } y = x^2 + 7x$$

$$= (y + 10)(y + 12) - 15$$

$$= y^2 + 12y + 10y + 120 - 15$$

$$= y^2 + 22y + 105$$

$$= y^2 + 7y + 15y + 105$$

$$\begin{aligned}
 &= y(y + 7) + 15(y + 7) \\
 &= (y + 7)(y + 15) \\
 &= (x^2 + 7x + 7)(x^2 + 7x + 15)
 \end{aligned}$$

(b) Simplify the following as a rational expression:

$$\frac{1}{x^2 - 8x + 15} + \frac{1}{x^2 - 4x + 3} - \frac{2}{x^2 - 6x + 5}$$

Ans
$$\frac{1}{x^2 - 8x + 15} + \frac{1}{x^2 - 4x + 3} - \frac{2}{x^2 - 6x + 5}$$

$$= \frac{1}{x^2 - 3x - 5x + 15} + \frac{1}{x^2 - x - 3x + 3} - \frac{2}{x^2 - x - 5x + 5}$$

$$= \frac{1}{x(x-3) - 5(x-3)} + \frac{1}{x(x-1) - 3(x-1)} - \frac{2}{x(x-1) - 5(x-1)}$$

$$= \frac{1}{(x-3)(x-5)} + \frac{1}{(x-1)(x-3)} - \frac{2}{(x-1)(x-5)}$$

$$= \frac{x-1 + x-5 - 2(x-3)}{(x-3) \cdot (x-1) \cdot (x-5)} = \frac{x-1 + x-5 - 2x + 6}{(x-3)(x-1)(x-5)}$$

$$= \frac{2x - 2x + 6 - 6}{(x-3)(x-1)(x-5)} = \frac{0}{(x-1)(x-3)(x-5)} = 0$$

Q.8.(a) Solve the inequality: $3x - 10 \leq 5 < x + 3$

Ans $3x - 10 \leq 5$, $5 < x + 3$

$3x \leq 5 + 10$, $5 - 3 < x$

$3x \leq 15$, $2 < x$

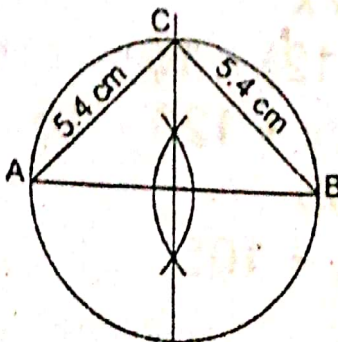
$x \leq \frac{15}{3}$

$x \leq 5$

$S.S = \{2 < x \leq 5\}$

(b) Construct a right angled isosceles triangle whose hypotenuse is 5.4 cm.

Ans



Q.9. Any point equidistant from the end points of a line segment is on the right bisector of it. (5)

Ans Given:

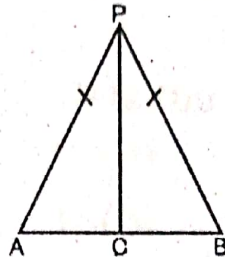
\overline{AB} is a line segment point P is such that $\overline{PA} \cong \overline{PB}$.

To prove:

The point P is on the right bisector of \overline{AB} .

Construction:

Join P to C the mid-point of \overline{AB} .



Proof:

Statements	Reasons
$\triangle ACP \leftrightarrow \triangle BCP$	
$\overline{PA} \cong \overline{PB}$	Given
$\overline{PC} \cong \overline{PC}$	Common
$\overline{AC} \cong \overline{BC}$	Construction
$\triangle ACP \cong \triangle BCP$	S.S.S \cong S.S.S
$\angle ACP \cong \angle BCP$... (i)	(Corresponding angles of congruent triangles)
But $m\angle ACP + m\angle BCP = 180^\circ$ (ii)	Supplementary angles
$m\angle ACP = m\angle BCP = 90^\circ$	From (i) and (ii)
$\overline{PC} \perp \overline{AB}$... (iii)	$m\angle ACP = 90^\circ$ (Proved)
Also $\overline{CA} \cong \overline{CB}$... (iv)	Construction
PC is a right bisector of \overline{AB}	From (iii) and (iv)
the point P is on the right bisector of \overline{AB} .	

OR

Triangles on the same base and of the same (i.e., equal) altitudes are equal in area.

Ans

For Answer see Paper 2014 (Group-I), Q.9.(OR).